2. Mathematical Models of Mechanical Systems

2.1. Introduction

A very simple class of mechanical system

A single rigid body or
of two rigid bodies simply connected } . . . Planar motion including rotating motion in the plane

Free-body diagram → Newton's and Euler's laws $\Longrightarrow$ the differential equations

The mathematical model . . . { A set of differential equations that describe the physical system and its environment, plus certain auxiliary information.

The model includes any approximations.[2]

Several severe restrictions on the general validity of our analytical approach $\longrightarrow$ Simplifying our models and yielding solutions to the equations that are accurate enough for many practical purposes.

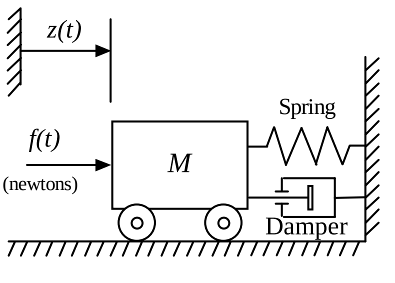
Simple mechanical sysmtems and the ciucumstances . . . Ordinary differential equations with constant coefficients

The valid time interval
The appropriate physical conditions
at the initial instant of that time interval } $\Rightarrow$ The initial-value problem on differential equations

The modeling process . . . The bridge between the physics and the mathematics of the problem

2.2. Mass, Spring, and Damper System

A simple mass, spring, and damper system as shown in Fig.2.1



Frictionless, massless wheels
Fig.2.1 The mass, spring, and damper system.

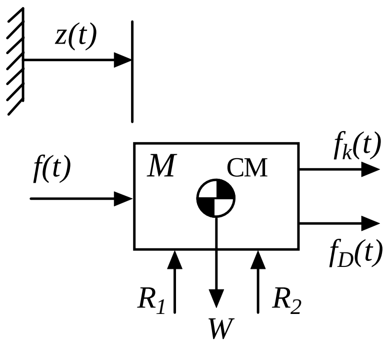


Fig.2.2 Free-body force-mass diagram.

$z(t)$	: The displacement of the cart from a reference point fixed to the level plane
$z = 0$	: The position in which the spring is relaxed.
The level plane	: An inertial reference frame The mass of the wheels is ignored. The friction in the wheel bearings is negligible. A force $f(t)$ is applied in the positive $z(t)$ direction.
M	: The mass of the cart and is constant.
The motion of the cart	: only in the horizontal direction, does not rotate, nor do the wheels lose contact with the plane

Newton's second law $\Longleftarrow$ The free-body force-mass diagram of Fig.2.2

[1]The boundary conditions exisiting between the system and its environment, the identification of the exciting agents(inputs) to the system, and the inteval of time during which the differential equations provide a valid description of the system dynamics.

[2]For exmaple, we will ignore the earth's rotation and assume that the earth is an inertial reference frame.

Five agents applying a force to the mass
Gravity, the vertical contact forces supporting the cart, the external driving force $f(t)$, the contact force due to the spring $f_k(t)$ and the contact force due to the damper plunger $f_D(t)$ From assumption that the cart does not accelerate in the vertical direction,

$$\therefore \quad R_1 + R_2 - W = 0$$

The two forces R_1 and R_2 also have moments about the center of mass.
From the assumption that the cart does not rotate, the sum of the moments will be zero.

$$\Downarrow$$

The terms R_1 and R_2 will fluctuate as the cart moves to and fro. $\Leftrightarrow$ the sum of the moments remains zero.

$\Longrightarrow$ Only the motion in the z direction

Newton's second law for motion, the rate of change of the linear momentum of the mass in the z direction is equal to the sum of all the force applied to the mass in the z direction

$$M\ddot{z} = f(t) + f_k(t) + f_D(t) \tag{2.1}$$

The unit for mass, M: kilograms[kg], the unit for displacement: meter [m], the unit of force: newtons [N], the unit of time: seconds [s].
$\ddot{z}(t)$: units of meter per second per second [(m/s)/s]
The spring force and the damper force . . . depending on the motion of the cart

the spring force	. . .	the displacement $z(t)$
the damper force	. . .	the relative velocity between the plunger and the cylinder in this case $\dot{z}(t)$

$\Longrightarrow$ the dependence on the physical properties of the spring and the damper
The simplest type of spring model . . . the ideal (lossless) Hooke's law

$$f_k(t) = -kz(t) \tag{2.2}$$

where k is the spring constant having units of N/m.

The minus sign: the arrow denoting $f_k(t)$ in Fig.2.2 points in the $+z$ direction
In the dampler: a viscous fluid working around the plunger as it moves back and forth in the cylinder transmits the force $f_D(t)$ to the cart.
A laboratory test of the damper . . . a force-velocity characteristic resembling that shown in Fig.2.3.
A model characteristic:

$$f_D(t) = -[\alpha\dot{z}(t) + \beta\dot{z}^3(t)] \tag{2.3}$$

where α N/(m/s) and β N/(m/s)^3 are coefficient chosen so that Eqn.2.3 will fit the laboratory data of Fig.2.3.

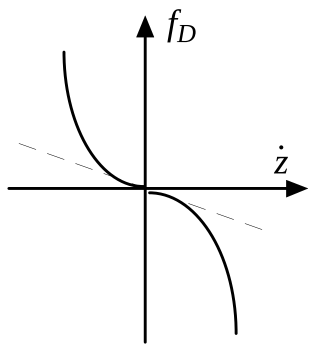


Fig.2.3 Force-velocity character of damper.

If mechanical contact exists between plunger and cylinder it will not create a friction force independent of $\dot{z}(t)$.
$z(t)$: 制限

$$M\ddot{z}(t) + \alpha\dot{z}(t) + \beta\dot{z}^3(t) + kz(t) = f(t) \tag{2.4}$$

$M \neq 0$

$$\ddot{z}(t) + \left[\frac{\alpha}{M}\right]\dot{z}(t) + \left[\frac{\beta}{M}\right]\dot{z}^3(t) + \left[\frac{k}{M}\right]z(t) = \left[\frac{1}{M}\right]f(t) \tag{2.5}$$

$\Longrightarrow$ 二階非線形常微分方程式 $\longrightarrow$ 連立の一階常微分方程式 ($z(t) = x_1(t)$, $\dot{z}(t) = x_2(t)$: 状態変数)

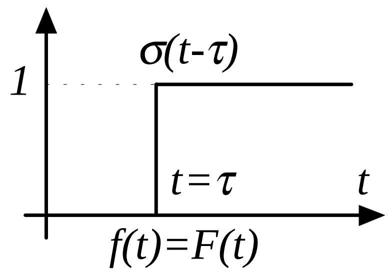
$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -\left[\tfrac{k}{M}\right]x_1(t) - \left[\tfrac{\alpha}{M}\right]x_2(t) - \left[\tfrac{\beta}{M}\right]x_2^3(t) + \left[\tfrac{1}{M}\right]f(t) \end{aligned} \tag{2.6}$$

初期値問題

$$
\begin{array}{lll}
t_0 \leqq t \leqq t_F & f(t) & \text{: a known(given) function} \\
t = t_0 & x_1(t_0), x_2(t_0) & \text{: given at the initial instant} \\
\text{Arbitrary } t & z_{min} \leqq z(t) \leqq z_{max} & \text{: A restricted variable}
\end{array}
$$

初期条件

$$
\begin{aligned}
x_1(0) &= z(0) \text{ [m]} \\
x_2(0) &= \dot{z}(0) \text{ [m/s]}
\end{aligned}
$$



Input function(Step)

カートの初期加速度の決定

$$\ddot{z}(0^+) = \dot{x}_2(0^+) = \frac{1}{M}[F - kz(0) - \alpha\dot{z}(0) - \beta[\dot{z}(0)]^3 \text{ m/s}^2$$

$$\Downarrow$$

$$z(t_F); \quad \begin{array}{l} \dot{x}_1(t_F) = 0 \\ \dot{x}_2(t_F) = 0 \end{array} \Longrightarrow z(t_F) = \frac{F}{k} \text{ [m]}$$

$\beta\dot{z}^3(t)$ 非線形項 $\Rightarrow z(t)$ に他の制限を付加 (運動に関して) . . . 速度が小さい
. . . カートの速度が十分小さくなる　$f_D(t) \cong -\alpha\dot{z}(t) \longrightarrow$ 点線 (図)

$$
\begin{aligned}
&\dot{x}_1(t) = x_2(t) \\
&\dot{x}_2(t) = -\left[\frac{k}{M}\right] x_1(t) - \left[\frac{\alpha}{M}\right] x_2(t) + \left[\frac{1}{M}\right] f(t) \\
&t_0 \leqq t \leqq t_F \text{ and } f(t) \text{ is known,} \\
&x_1(t_0) \text{ and } x_2(t_0) \text{ are known,} \\
&z_{min} \leqq x_1(t) \leqq z_{max} \text{ and } \beta\dot{z}^3(t) \cong 0
\end{aligned}
\tag{2.9}
$$

2

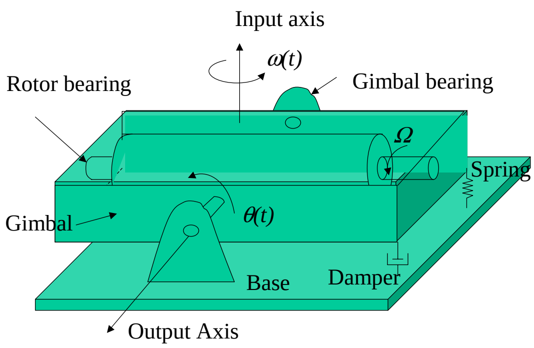


Fig.2.4 Simple rate gyroscope

ジャイロスコープ :
センサとして使用 (航空機，宇宙船等)
　$\downarrow\rightarrow$ 天文台の Bose(?)
位置，速度，加速度を与える？ . . . 機械的
レートジャイロスコープ :
移動台の角度割合に比例した信号を与える
ジンバル . . . 箱の回転角 $\theta(t)$
回転 0.1 rad $\longrightarrow \frac{0.1}{2\pi} \times 360 = 6^0$ を超えない
　. . . 小さいと見なす
$$\Downarrow$$
$\omega(t)$ を測定する system
$\boldsymbol{k}$ 軸 . . . 出力軸と同じ軸
ジンバル . . . 慣性空間に関連
単位ベクトル $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ を定める

角運動量

$$\hat{H}(t) = I_R\Omega\hat{\boldsymbol{i}}(t) + I_0\dot{\theta}(t)\hat{\boldsymbol{k}}(t) \text{ kg} \cdot \text{m/s}^2 \tag{2.10}$$

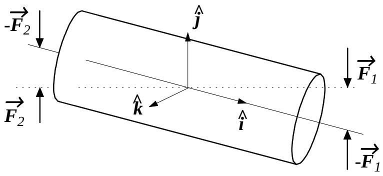
I_R : $\hat{\boldsymbol{i}}$軸についての慣性モーメント kgm^2
I_0 : $\hat{\boldsymbol{k}}$軸についての慣性モーメント kgm^2
Ω : ロータの回転速度 (角速度) rad/s
　$\rightarrow$ 電気モータ等で一定回転を与える

$I_R\Omega \rightarrow$ 大きくする
　$\Omega >> \omega(t) \rightarrow$ 式 (2.10) において $\omega(t)$ を無視
オイラーの法則において，力がロータに作用しない場合
　$\hat{H}(t)$: 慣性空間において一定（保存）
$\longrightarrow$ ジャイロスコープの基本特性の調査
オイラーの法則 :
　ロータに作用する任意力とロータの合成運動との関係を与える
モーメントの総和

$$\overrightarrow{M}(t) = \frac{d\hat{H}(t)}{dt} \text{ N-m} \tag{2.11}$$

従って，

$$\overrightarrow{M}(t) = \frac{d\hat{H}(t)}{dt} = I_P\Omega\frac{d\boldsymbol{i}(t)}{dt} + I_0\ddot{\theta}(t)\hat{\boldsymbol{k}}(t) + I_0\dot{\theta}(t)\frac{d\hat{\boldsymbol{k}}}{dt} \tag{2.12}$$



Unit vectors on a gyroscope.

角運動量

$$\overrightarrow{H}(t) = I_R\Omega\hat{\boldsymbol{i}}(t) + I_0\theta(t)\hat{\boldsymbol{k}}(t) \text{ kg-m}^2\text{/s} \tag{2.10}$$

から

$$\overrightarrow{M}(t) = \frac{d\hat{H}(t)}{dt} \text{ N-m}$$

$$= I_R\Omega\frac{d\hat{\boldsymbol{i}}(t)}{dt} + I_0\ddot{\theta}(t)\hat{\boldsymbol{k}}(t) + I_0\dot{\theta}(t)\frac{d\hat{\boldsymbol{k}}(t)}{dt} \tag{2.12}$$

ここで，I_R: i 軸，I_0; k 軸回りの慣性モーメント
ジャイロスコープが角速度 ω を受けたとすると

$$\overrightarrow{\boldsymbol{\varpi}}(t) = \omega t\hat{\boldsymbol{j}} \text{ [rad/s]} \tag{2.13}$$

また，

$$\frac{d\boldsymbol{i}(t)}{dt} = -\omega(t)\hat{\boldsymbol{k}}(t)$$
$$\frac{d\boldsymbol{k}(t)}{dt} = \omega(t)\hat{\boldsymbol{k}}(t)$$

の関係が成立しているので，この式を式 (2.12) に代入すると

$$\overrightarrow{H}(t) = [I_0\dot{\theta}(t) - I_R\Omega\omega(t)]\hat{\boldsymbol{k}}(t) \text{ N-m} \tag{2.14}$$

ここで

$$\dot{\theta}(t)\omega(t) = 0(<< \Omega\omega(t))$$

さらに，I_G をジンバル箱の慣性モーメントして

$$I_G\ddot{\theta}(t) = I_R\Omega\omega(t) - I_0\ddot{\theta}(t) - k\theta(t) - d\dot{\theta}(t)$$

すなわち，ジンバルにかかる回転モーメント $M(t)$ は,

$$I_G\ddot{\theta}(t) + k\theta(t) + d\dot{\theta}(t) = -M(t)$$

よって，

$$[I_G + I_0]\ddot{\theta}(t) + d\dot{\theta}(t) + k\theta(t) = [I_R\Omega]\omega(t) \tag{2.15}$$

ここで，k: N-m/rad，d: N-m/(rad/s).
エンコーダー，ポテンショメーター
$\Longrightarrow$ 回転角度，回転速度測定
$\Longrightarrow$ 出力 output : 電圧
出力 $\Longrightarrow$ 電圧 [V] として出てくる．すなわち

$$e_{out} = K_s\theta(t) \text{ [V]} \tag{2.16}$$

K_s: Scale factor 比例定数 [V/rad]
定常応答 $\omega(t) = \omega_{ss}$(一定)

$$\ddot{\theta}(t) = 0, \quad \dot{\theta}(t) = 0$$
$$\theta_{ss} = [I_R\Omega/k]\,\omega_{ss}$$

式 (2.16) に代入

$$e_{out} = K_s\left[\frac{I_R\Omega}{k}\right]\omega_{ss}$$

$$\therefore \quad \frac{e_{out}}{\omega_{ss}} = \frac{K_sI_R\Omega}{k} \text{ [V/rad/s]} \tag{2.17}$$

$$\begin{array}{l} x_1(t) = \theta(t) \\ x_2(t) = \dot{\theta}(t) \end{array} \longrightarrow I_T = I_G + I_0$$

したがって，次の状態方程式モデルを得る．

$$\begin{array}{rl} \text{Dynamics:} & \dot{x}_1(t) = x_2(t) \\ & \dot{x}_2(t) = -\left[\dfrac{k}{I_T}\right]x_1(t) - \left[\dfrac{d}{I_T}\right]x_2(t) + \left[\dfrac{I_R\Omega}{I_T}\right]\omega(t) \\ \text{Sensor:} & e_{out}(t) = K_s\theta(t) \\ \text{Conditions:} & t_0 \leqq t \leqq t_F;\, x_1(t_0)\,, \text{ and } \omega(t) \text{ are known.} \end{array} \tag{2.18}$$

2.4. Pendulum with Moving Base

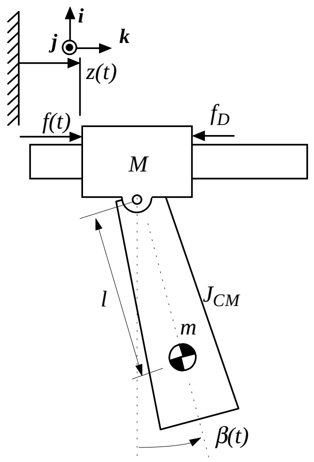


Fig2.6 Cart-pendulum system.

図 2.6 に示すように，動く台車に取り付けられた振り子の運動を考える．図に示すように直交座標系に対応した単位ベクトルを $\boldsymbol{i}$，$\boldsymbol{j}$，$\boldsymbol{k}$ とする．
台車の運動は水平方向のみに固定されているとし，質量を M，基準点からの変位を $z(t)$，作用する力を $f(t)$ とする．レールと台車の間には，粘性摩擦 $\vec{f}_D$ が作用していると仮定する．また，振り子の質量を m，$\boldsymbol{j}$ 軸回りの慣性モーメントを J_{CM}，回転中心から重心までの距離を l とし，垂直下向きにおろした線からの反時計回りの回転角を $\beta(t)$ とする．

$$\Downarrow$$

Free-body diagram ⟶ Fig.2.7

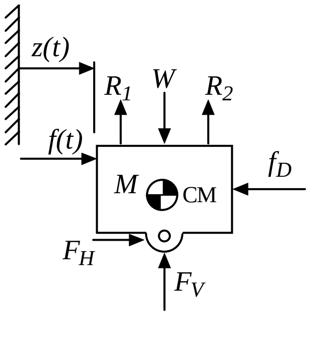


Fig2.7a Free-body diagram for the cart.

ここで，

$$\vec{W} = -Mg\hat{\boldsymbol{i}}$$

ここで，R_1，R_2，$\vec{F}_V$: レールとカートの間に働く反力とし，

$$\vec{f}(t) = f(t)\hat{\boldsymbol{k}}$$
$$\vec{f}(t) = -D\dot{z}(t)\hat{\boldsymbol{k}}$$

及び振り子からの反力を $\vec{F}_H$ とすると，カートの運動方程式は

$$M\ddot{z}(t) = f(t) - D\dot{z}(t) + F_H(t) \qquad (2.19)$$

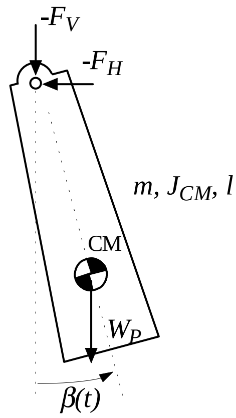


Fig2.7b Free-body diagram for the pendulum.

振り子に対する Free-body diagram において，$\vec{W}_P = mg\cdot(-\boldsymbol{i})$ であり，さらに，空気摩擦をを無視すると，Newton の第２法則より，振り子の運動方程式は

$$\vec{W}_P - \vec{F}_V(t) - \vec{F}_H(t) = mg(-\hat{\boldsymbol{i}}) - F_V\boldsymbol{i} - F_H(t)\hat{\boldsymbol{k}} \qquad (2.21)$$

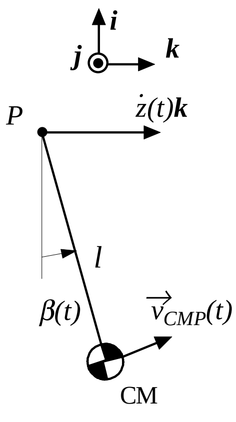


Fig2.8 Kinematics of pendulum motion.

また，図 2.8 より

$$\hat{\boldsymbol{v}}_{CMI}(t) = \dot{z}(t)\hat{\boldsymbol{k}} + \vec{v}_{CMP}(t) \qquad (2.22)$$

成分で表すと

$$\boldsymbol{v}_{CMI} = \left[l\dot{\beta}(t)\sin\beta(t)\right]\hat{\boldsymbol{i}} + \left[\dot{z}(t) + l\dot{\beta}(t)\cos\beta(t)\right]\hat{\boldsymbol{k}} \qquad (2.23)$$

従って，質量中心の加速度は

$$\hat{\boldsymbol{a}}_{CMI}(t) = \frac{d\hat{\boldsymbol{v}}_{CMI}(t)}{dt} = \left[l\ddot{\beta}(t)\sin\beta(t) + l\dot{\beta}^2(t)\cos\beta(t)\right]\hat{\boldsymbol{i}} + \left[\ddot{z}(t) + l\ddot{\beta}(t)\cos\beta(t) - l\dot{\beta}^2(t)\sin\beta(t)\right]\hat{\boldsymbol{k}} \tag{2.25}$$

例えば，Euler の法則をこの運動に対して表現すると

$$\begin{bmatrix} \text{sum of all moments of all forces} \\ \text{acting on the body about an axis} \\ \text{through the center of mass and} \\ \text{normal to the plane of motion} \end{bmatrix} = J_{CM} \times \begin{bmatrix} \text{angular acceleration} \\ \text{of the body with} \\ \text{respect to inertial space} \end{bmatrix} \tag{2.26}$$

ここで，J_{CM} は重心回りの振り子の慣性モーメント．重心回りの運動を考えると $\overrightarrow{W}_P$ によるモーメントは零，回転軸における接触力によるモーメントの合計は

$$\left[lF_H\left(t\right)\cos\beta\left(t\right) + lF_V\left(t\right)\sin\beta\left(t\right)\right]\overrightarrow{j} \tag{2.27}$$

式 (2.26) の右辺は

$$J_{CM}\ddot{\beta}\left(t\right)\overrightarrow{j} \tag{2.28}$$

従って，式 (2.21), (2,25) から $\overrightarrow{i}$ 方向, $\overrightarrow{k}$ 方向に対応するスカラー方程式が次のように定まる．

$$ml\left[\ddot{\beta}(t)\sin\beta(t) + \dot{\beta}^2(t)\cos\beta(t)\right] = -F_V\left(t\right) - mg, \tag{2.29}$$

$$m\ddot{z}(t) + ml\ddot{\beta}(t)\cos\beta(t) - ml\dot{\beta}^2(t)\sin\beta(t) = -F_H\left(t\right). \tag{2.30}$$

さらに，式 (2.27), (2.28) から

$$lF_H\left(t\right)\cos\beta\left(t\right) + lF_V\left(t\right)\sin\beta\left(t\right) = J_{CM}\ddot{\beta}\left(t\right). \tag{2.31}$$

また，式 (2.19)，(2.31) から

$$\left[M + m\right]\ddot{z}(t) + \left[ml\cos\beta\left(t\right)\right]\ddot{\beta}\left(t\right) = f\left(t\right) - D\dot{z}\left(t\right) + ml\dot{\beta}^2(t)\sin\beta(t), \tag{2.32}$$

$$\left[ml\cos\beta\left(t\right)\right]\ddot{z}(t) + \left[J_{CM} + ml^2\right]\ddot{\beta}(t) = -mgl\sin\beta\left(t\right). \tag{2.33}$$

それぞれ，$\ddot{z}\left(t\right), \ddot{\beta}\left(t\right)$ で表現する形式に変形する．

$$\ddot{z}\left(t\right) = \frac{1}{N}\left\{\left(J_{CM} + ml^2\right)\left[f\left(t\right) - D\dot{z}\left(t\right) + ml\dot{\beta}^2(t)\sin\beta(t)\right] + \left(ml^2\right)g\cos\beta\left(t\right)\sin\beta\left(t\right)\right\}, \tag{2.34}$$

$$\ddot{\beta}\left(t\right) = -\frac{ml}{N}\left\{\left(M + m\right)g\sin\beta\left(t\right) + ml\dot{\beta}^2\left(t\right)\cos\beta\left(t\right)\sin\beta\left(t\right) + \cos\beta\left(t\right)\left[f\left(t\right) - D\dot{z}\left(t\right)\right]\right\} \tag{2.35}$$

ここで,

$$N = \left(M + m\right)J_{CM} + ml^2\left(M + m\sin^2\beta\left(t\right)\right). \tag{2.36}$$

状態変数形式に変換するために次の状態変数を考える．

$$x_1\left(t\right) = z\left(t\right), x_2\left(t\right) = \dot{z}\left(t\right), x_3\left(t\right) = \beta\left(t\right), \text{ and } x_4\left(t\right) = \dot{\beta}\left(t\right). \tag{2.37}$$

したがって,

$$\begin{aligned} \dot{x}_1\left(t\right) &= x_2\left(t\right), \\ \dot{x}_2\left(t\right) &= a_{22}x_2\left(t\right) + a_{23}\sin x_3\left(t\right) + a_{2NL}\left(t\right) + b_2 f\left(t\right), \\ \dot{x}_3\left(t\right) &= x_4\left(t\right), \\ \dot{x}_4\left(t\right) &= a_{42}x_2\left(t\right) + a_{43}\sin x_3\left(t\right) + a_{4NL}\left(t\right) + b_4 f\left(t\right). \end{aligned} \tag{2.38}$$

ここで,

$$\begin{aligned} &a_{22} = -\left[\frac{D\left(J_{CM}+ml^2\right)}{N}\right], \quad a_{23} = \left[\frac{\left(ml^2\right)g\cos x_3(t)}{N}\right] \quad b_2 = \left[\frac{J_{CM}+ml^2}{N}\right], \\ &a_{42} = \left[\frac{mlD\cos x_3(t)}{N}\right], \quad a_{43} = -\left[\frac{mlg(M+m)}{N}\right], \quad b_4 = -\left[\frac{ml\cos x_3(t)}{N}\right], \\ &a_{2NL} = \left[\frac{\left(J_{CM}+ml^2\right)mlx_4^2(t)\sin x_3(t)}{N}\right], \\ &a_{4NL} = -\left[\frac{(ml)^2x_4^2(t)\cos x_3(t)\sin x_3(t)}{N}\right], . \end{aligned} \tag{2.39}$$

一般には，このように非線形項を含む方程式となる．そこで，通常行われているように，微小運動を仮定して，$\beta(t) << 1$，$\cos\beta(t) \approx 1, \sin\beta(t) = \beta(t)$ と仮定し，$\dot{\beta}^2(t)\sin\beta(t) << 1$ などの変形を行う．すなわち，次のような線形な状態変数方程式の組が得られる．

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t), \\
\dot{x}_2(t) &= a_{22}x_2(t) + a_{23}x_3(t) + b_2 f(t), \\
\dot{x}_3(t) &= x_4(t), \\
\dot{x}_4(t) &= a_{42}x_2(t) + a_{43}x_3(t) + b_4 f(t).
\end{aligned} \tag{2.40}$$

ここで,

$$\begin{aligned}
a_{22} &= -\left[\frac{D\left(J_{CM}+ml^2\right)}{(M+m)J_{CM}+mMl^2}\right], & a_{23} &= \left[\frac{\left(ml^2\right)g}{(M+m)J_{CM}+mMl^2}\right], \\
b_2 &= \left[\frac{J_{CM}+ml^2}{(M+m)J_{CM}+mMl^2}\right], & a_{42} &= \left[\frac{mlD}{(M+m)J_{CM}+mMl^2}\right], \\
a_{43} &= -\left[\frac{mlg(M+m)}{(M+m)J_{CM}+mMl^2}\right], & b_4 &= -\left[\frac{ml}{(M+m)J_{CM}+mMl^2}\right],
\end{aligned} \tag{2.41}$$

すなわち，線形の連立一階（常）微分方程式であり，行列形式で表わすと

$$\underbrace{\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix}}_{4\times 1} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & a_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & a_{43} & 0 \end{bmatrix}}_{4\times 4} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}}_{4\times 1} + \underbrace{\begin{bmatrix} 0 \\ b_2 \\ 0 \\ b_4 \end{bmatrix}}_{4\times 1} \underbrace{[f(t)]}_{1\times 1}. \tag{2.42}$$

よって，次にような状態方程式が求まる．

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}f(t). \tag{2.43}$$

ここで,

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix}, \boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}, \boldsymbol{A} = \{a_{ij}\} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & a_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & a_{43} & 0 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 0 \\ b_2 \\ 0 \\ b_4 \end{bmatrix}. \tag{2.44}$$

$\dot{\boldsymbol{x}}(t), \boldsymbol{x}(t) : 4\times 1$ 行列，あるいは，４次元ベクトル空間.
$\boldsymbol{A}$: ４次元ベクトル空間 $\boldsymbol{x}(t)$ から４次元ベクトル空間 $\dot{\boldsymbol{x}}(t)$ への変換行列.
$\boldsymbol{B}$: １次元ベクトル空間 $f(t)$ から４次元ベクトル空間 $\dot{\boldsymbol{x}}(t)$ への変換行列.